MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2016/2017 SESSION

PTG 0116 – TRIGONOMETRY AND COORDINATE GEOMETRY

(All sections / Groups)

10 OCTOBER 2016
9.00 a.m. – 11.00 a.m.
(2 Hours)

INSTRUCTIONS TO STUDENTS

1. This question paper consists of 3 pages with 4 questions and an appendix.
2. Answer all questions.
3. Unless stated otherwise, if an answer is given as a decimal, it should be rounded to **four** significant figures.
4. Write your answers in the Answer Booklet provided.
5. Show all relevant steps to obtain maximum marks.

Question 1

(a) Derive the identity $\tan^2\theta+1=\sec^2\theta$. [4 marks]

(b) Without using a calculator, find the exact values of $\sec\left[\tan^{-1}\frac{12}{5}\right]$ and $\sin\left[\tan^{-1}\frac{12}{5}\right]$. [6 marks]

(c) Solve $2\sin\theta-\cos^2\theta=\sin^2\theta$ for $0\le\theta\le3\pi$. [6 marks]

(d) Solve the triangle in Figure 1 below. Note that the figure is not to scale. [9 marks]

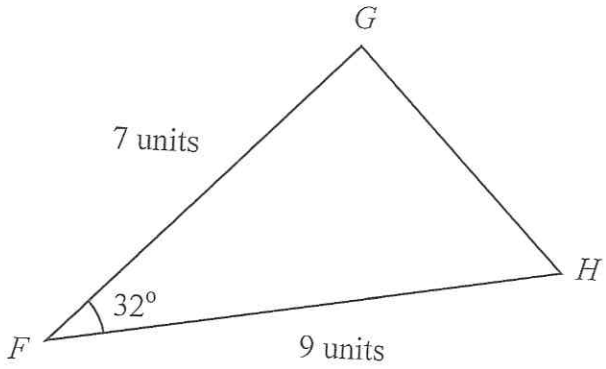

Figure 1

Question 2

(a) Let $a=-10+i$ and $b=5-2i$. Find the given complex expression in standard form.

(i) $5a-2b$, [2 marks]

(ii) $\frac{b^2}{a}$. [4 marks]

(b) The polar coordinates of point P are $(-3, \pi/3)$. Express the same point in polar coordinates with

(i) $r>0$ and $0\le\theta<2\pi$, [1 mark]

(ii) $r<0$ and $-2\pi<\theta\le0$. [1 mark]

(c) Convert $-2+9i$ into polar form. Express the argument in degrees. [5 marks]

(d) Suppose that $\underset{\sim}{a}=-3\underset{\sim}{i}+3\underset{\sim}{j}+\underset{\sim}{k}$ and $\underset{\sim}{b}=3\underset{\sim}{i}+\underset{\sim}{j}-2\underset{\sim}{k}$ are vectors in three-dimensional space.

(i) Decompose $\underset{\sim}{b}$ into $\underset{\sim}{b}_1$ and $\underset{\sim}{b}_2$ where $\underset{\sim}{b}_1$ is parallel to $\underset{\sim}{a}$ and $\underset{\sim}{b}_2$ is orthogonal to $\underset{\sim}{a}$. [6 marks]

(ii) Show that $\underset{\sim}{b}_1$ and $\underset{\sim}{b}_2$ are orthogonal. [3 marks]

(iii) Find the vector whose length is 3 units and in the opposite direction of $\underset{\sim}{a}$. [3 marks]

Continued...

Question 3

(a) Find the equation of a line that is perpendicular to the line $y = -64$ and contains the point $(-7, 2)$. [1 mark]

(b) Given the line L passes through points $A\ (5, 7)$ and $B\ (23, -2)$.
(i) Find the equation of line L in slope-intercept form. [3 marks]
(ii) Determine if point $C\ (-1, 10)$ is on line L. [2 marks]

(c) Find the centre, radius, and x- and y-intercepts of the circle $x^2 + y^2 - 8x + 10y - 8 = 0$. Sketch the circle. [10 marks]

(d) The equation of an ellipse is $\frac{(x+1)^2}{25} + \frac{(y-3)^2}{9} = 1$. Find the coordinates of the centre, foci and vertices of the ellipse. [9 marks]

Question 4

(a) Let $A = \begin{bmatrix} 1 & 10 \\ -1 & 2 \\ 3 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 & -1 \\ 2 & -2 & 0 \\ 5 & 3 & 4 \end{bmatrix}$. Compute the following matrix expression if it is defined.

(i) $A + B$, [1 mark]
(ii) AB, [1 mark]
(iii) BA, [2 marks]
(iv) $A^{\mathrm{T}}B$, [3 marks]
(v) $A + (A^{\mathrm{T}}B)^{\mathrm{T}}$. [2 marks]

(b) Let $P = \begin{bmatrix} 1 & 3 & 3 \\ -2 & -1 & -1 \\ -6 & -13 & -8 \end{bmatrix}$ and $Q = \begin{bmatrix} 1 & 3 & 0 \\ 2 & -2 & 1 \\ -4 & 1 & -1 \end{bmatrix}$.

(i) Determine if Q is the inverse of P. [3 marks]
(ii) Find the inverse of P. [3 marks]
(iii) Find the inverse of Q. [3 marks]

(c) Consider the following system of linear equations:

$$x + 3y = 5$$
$$2x - 2y + z = -5$$
$$-4x + y - z = 10$$

(i) Write the system as a matrix equation. [1 mark]
(ii) By using the results obtained in part (b), solve for x, y, and z. [6 marks]

Continued...

APPENDIX

$$\sin^2\theta+\cos^2\theta=1$$
$$\tan^2\theta+1=\sec^2\theta$$
$$1+\cot^2\theta=\csc^2\theta$$

$$\cos(A+B)=\cos A\cos B-\sin A\sin B$$
$$\cos(A-B)=\cos A\cos B+\sin A\sin B$$
$$\sin(A+B)=\sin A\cos B+\cos A\sin B$$
$$\sin(A-B)=\sin A\cos B-\cos A\sin B$$
$$\tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\tan B}$$
$$\tan(A-B)=\frac{\tan A-\tan B}{1+\tan A\tan B}$$

$$\sin 2A=2\sin A\cos A$$
$$\cos 2A=\cos^2 A-\sin^2 A$$
$$=2\cos^2 A-1$$
$$=1-2\sin^2 A$$
$$\tan 2A=\frac{2\tan A}{1-\tan^2 A}$$

$$\sin^2 A=\frac{1-\cos 2A}{2}$$
$$\cos^2 A=\frac{1+\cos 2A}{2}$$
$$\tan^2 A=\frac{1-\cos 2A}{1+\cos 2A}$$
$$\tan A=\frac{\sin 2A}{1+\cos 2A}=\frac{1-\cos 2A}{\sin 2A}$$

$$\sin A\cos B=\frac{1}{2}\left[\sin(A-B)+\sin(A+B)\right]$$
$$\cos A\cos B=\frac{1}{2}\left[\cos(A-B)+\cos(A+B)\right]$$
$$\sin A\sin B=\frac{1}{2}\left[\cos(A-B)-\cos(A+B)\right]$$

$$\sin A+\sin B=2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$$
$$\sin A-\sin B=2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$$
$$\cos A+\cos B=2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$$
$$\cos A-\cos B=-2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$$

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